Astroparticle Physics

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You don't have to use separate sheets for every question.
Write your name and S number on every sheet
There are **5 questions** with a total number of marks: 55

WRITE CLEARLY

(1) (Total 14 marks)
The Friedman equation is given as:

$$H^2 = \left[\frac{\dot{R}}{R}\right]^2 = \frac{8\pi\ G\ \rho_{tot}}{3} - \frac{k\ c^2}{R^2}$$

where R is the scale parameter of the Universe. In addition, there is the fluid equation,

$$\dot{\rho}c^2 + 3\frac{\dot{R}}{R}\left(\rho c^2 + P\right) = 0$$

Finally, the equation of state couples the pressure P to the density ρ through the parameter w:

$$P = w\ \rho c^2$$

where w is equal to 0, 1/3, and -1, for the matter, radiation, and vacuum dominated densities, respectively.

(a) (2 marks)
The Taylor expansion of $\frac{R(t)}{R(0)}$ at $t = 0$ up to second order in terms of $[H_0 \times (t - t_0)]$ can be written as:

$$\frac{R(t)}{R(0)} = 1 + \alpha \times H_0 \times (t - t_0) + \frac{1}{2}\beta \times \left[H_0 \times (t - t_0)\right]^2,$$

where t_0 indicates the time of the Big Bang ($t = 0 = t_0$). The second-order term of this expansion is known as the deceleration parameter $q(t)$. Prove that this deceleration parameter can be written as:

$$q = -\beta = -\frac{\ddot{R}\ R}{\dot{R}^2}$$

The parameter q describes what the fate of the Universe will be in the lang run, if q is positive the Universe will eventually collapse again, if it is negative the expansion of the Universe will increase rapidly, the Universe will go into a "big rip": even stars and atoms will break apart.

(b) (6 marks)
Using the fluid equation, and the equation of state, proof that the deceleration parameter can also be written as:

$$q = \left[\frac{4\pi\ G}{3c^2H^2}\right]\left[\rho c^2 + 3P\right]$$

(c) (2 marks)
Which value for w is needed to have a Universe where $q = 0$?

(d) (4 marks)
Express the deceleration parameter q in terms of dimensionless energy densities for radiation, matter, and vacuum: Ω_r, Ω_m and Ω_Λ where $\Omega = \rho/\rho_c$ and $\rho_c = \frac{3H^2}{8\pi G}$.

(2) (Total 16 marks)
When an ultra-high-energy proton arrives at the top of the atmosphere a particle cascade (air shower) develops. After a number of cascades the creation of new particles stops because the energy of the particles in the shower drops below a certain value, which we know as the critical energy E_c.

Assume for the present exercise the following:
The value for the critical energy $E_c = 80$ MeV.
Electron mass $m_e c^2 = 511\,\text{keV}$; Muon mass $m_\mu c^2 = 106\,\text{MeV}$; Pion mass about $m_\pi c^2 = 140\,\text{MeV}$ (forget that the pion mass depends a bit on the quarks involved); Proton mass: $m_p c^2 = 0.938\,\text{GeV}$.
Life time charged pions: 2.6×10^{-8} s; life time neutral pion: 8.4×10^{-17} s.
The relation between the penetration depth x into the atmosphere and the height h above sea level is given as $x = x_0 \exp[-h/H]$, with $x_0 = 1030$ g cm^{-2} and $H = 6.5$ km.
The index of refraction n of air at sea level is $n = 1 + 2.7 \times 10^{-4}$.
The density ρ of air at sea level is $\rho = 1.2 \times 10^{-3}$ g cm^{-3}.
The hadronic interaction length $\lambda_{int} = 100$ g cm^{-2} and the radiation length for photons and electrons/positrons is $\lambda_{rad} = 37$ g cm^{-2}.

(a) (2 marks)
What processes determine the actual value of the critical energy?

(b) (2 marks)
Give a very short explanation why initially the energy in the electromagnetic part (electrons, positrons, and photons) of the air shower increases at the expense of the energy in the hadronic part.

(c) (4 marks)
Assume that in the cascade at a height of 10 km a charged pion π^+ of energy 10^{10} eV is produced. Give the mean distance (in g cm^{-2}) for the pion to decay (neglecting interactions) and compare this with the mean distance for the particle to interact (neglecting decay). Do the same for a neutral pion (π^0).

(d) (4 marks)
Calculate whether the charged pion will emit Cherenkov radiation.

(e) (4 marks)
Assume a photon of energy 5×10^9 eV is created at a height of 10 km and starts an electromagnetic shower. Calculate the penetration depth X_{max} (in g cm^{-2}) where the shower reaches its maximum and the number of particles at this maximum.

(3) (Total 8 marks)
Cosmic rays entering the atmosphere of the Earth can produce neutrinos.

(a) (2 marks)
Describe the mechanism behind this atmospheric neutrino creation process.

(b) (2 marks)
In Figure 1 you see the calculated ratio of the fluxes for muon and electron neutrinos as a function of the neutrino energy. This plot was made assuming that cosmic rays

enter from the zenith into the atmosphere and has been calculated for detection at sea level. Explain why this ratio increases as a function of the neutrino energy.

(c) (4 marks)
What will happen with this flux if the inclination angle with respect to the zenith (also known as the polar angle θ) will increase? Explain your answer and provide information which parameters are important to understand the outcome as a function of the polar angle θ.

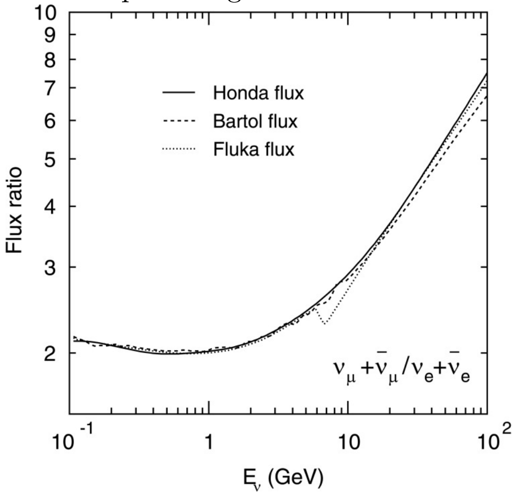

FIGURE 1. The calculated $(\nu_\mu + \overline{\nu}_\mu)/(\nu_e + \overline{\nu}_e)$ ratio of the atmospheric neutrino flux calculated at sea level and for cosmic rays entering vertically into the atmosphere as a function of the neutrino energy by three independent groups

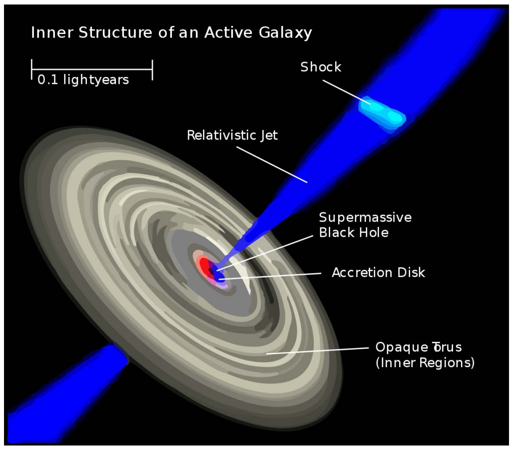

FIGURE 2. A schematic representation of the mass flow towards and from a super-massive black hole.

(4) (Total 11 marks)
Charged particles can be accelerated in case there are magnetic fields at the acceleration site. During the acceleration process these charged particles interact with the interstellar of even intergalactic medium.

(a) (2 marks)
Enrico Fermi coined an acceleration process, which requires the action of magnetic fields. Give two different examples of the Fermi acceleration process.

(b) (2 marks)
Give the main difference between these two processes, and explain in a few lines why they are different.

(c) (4 marks)
Of course, in case they are injected into the accelerator, both protons and electrons can be accelerated. How can you infer from measurements, whether electrons or protons (or both) are being accelerated. In your answer, provide an explanation for the origin of the observational evidence (i.e. the messenger) which relates to electron and/or proton acceleration.

(d) (3 marks)
In Figure 2 you see a schematic picture of a spinning black hole. Use this schematic picture to provide more insight into the previous sub-question. I.e. where do you think, that the relevant processes can take place? Explain why.

(5) (Total 6 marks)
Nuclear fusion evolves in stars through a certain sequence.

(a) (2 marks)
During a long period of the life of a star, which has a mass similar to that of the Sun, fusion is a very stable process. Why is that the case? Provide the names and the actions of the essential ingredients to maintain this long period of stability.

(b) (2 marks)
Which two physics process determine the position of the Gamow window as a function of the relative energy between two colliding nuclei?

(c) (2 marks)
Describe in a few lines the reasons why a heavy star (mass larger than 10 Solar masses) will collaps.

1a

$$\frac{R(t)}{R(0)} = 1 + \frac{\dot{R}(t)}{R(0)}(t-t_0) + \frac{1}{2}\frac{\ddot{R}(0)}{R(0)}(t-t_0)^2 + \dots$$

$$= 1 + H_0(t-t_0) + \frac{1}{2}\frac{\ddot{R}(0)}{R(0)} H_0^2 \frac{R^2(0)}{\dot{R}^2(0)}(t-t_0)^2 + \dots$$

The 2^{nd} coeff is: $\frac{\ddot{R}(0)}{\dot{R}^2(0)} R(0) = +\beta$ q.e.d.

1b Fluid equation:

$$\dot{\rho}c^2 + 3\frac{\dot{R}}{R}(\rho c^2 + P) = 0.$$

Friedman equation:

$$\frac{\dot{R}^2}{R^2} = H^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{R^2}$$

multiply with R^2

$$\dot{R}^2 + kc^2 = \frac{8\pi}{3}G\rho R^2$$

take time derivative: $2\dot{R}\ddot{R} = \frac{8\pi}{3}G\dot{\rho}R^2 + \frac{16\pi}{3}\rho G\dot{R}R$

Insert fluid equation ~~[illegible]~~ and divide by $2\dot{R}$ gives

pto

$$\ddot{R} = \frac{4\pi}{3}\left[-\frac{3}{c^2}\frac{\;}{R}(\rho c^2 + P)\right] G R^2 + \frac{8\pi}{3} G R \rho$$

$$= -4\pi R \rho G - \frac{4\pi P}{c^2} G R + \frac{8\pi}{3} G R \rho$$

$$= -\frac{4\pi G R}{3}\left(\rho + \frac{3P}{c^2}\right)$$

$$q = -\beta = -\frac{\ddot{R} R}{\dot{R}^2} = \frac{4\pi G}{3}\frac{R^2}{\dot{R}^2}\left(\rho + \frac{3P}{c^2}\right)$$

$$= \frac{4\pi G}{3c^2}\frac{1}{H^2}\left(\rho c^2 + 3P\right) \quad \text{qed}$$

1c $\quad \rho c^2 + 3P = 0.$

$$P = w \rho c^2$$

$$\rho c^2 + 3 w \rho c^2 = 0$$

$$(1 + 3w)\rho c^2 = 0$$

$$3w + 1 = 0 \qquad w = -1/3$$

1d $\quad \rho_c = \dfrac{3H^2}{8\pi G}$

$$q = \frac{\rho}{2\rho_c} + \frac{3P}{2\rho_c c^2} = \frac{\rho}{2\rho_c} + \frac{3}{2} w \frac{\rho}{\rho_c}$$

$$= \frac{\rho}{\rho_c}\left[\frac{1}{2} + \frac{3}{2} w\right]$$

$$= \frac{\rho_m}{\rho_c}\left[\frac{1}{2}+0\right] + \frac{\rho_r}{\rho_c}\left[\frac{1}{2}+\frac{3}{2}\frac{1}{3}\right] + \frac{\rho_\Lambda}{\rho_c}\left[\frac{1}{2}-\frac{3}{2}\right]$$

$$= \frac{1}{2}\Omega_m + \Omega_r - \Omega_\Lambda$$

2a Ionization losses compared to radiative losses

2b You create pions in the collision between hadron and hadrons. About 33% of these pions are neutral; they feed the electromagnetic channel.
By doing so the energy in the EM channel increases at the cost of the hadronic channel

2c π^+ life time $2.6\ 10^{-8}$ s.

$$d_{decay} = \gamma c \tau = \frac{E_\pi}{m_\pi c^2} c\tau = \frac{10^{10}\ 3.10^{8}\ 2.6\ 10^{-8}}{140.10^{6}}$$

$$= 557 \text{ m}.$$

$$\rho/_{10 km} = \rho_0 e^{-10/6.5} = 2.6\ 10^{-4} \text{ g/cm}^3$$

$$t = d * \rho = 557\ 10^2 * 2.6\ 10^{-4} = 14 \text{ g/cm}^2$$

$d_{decay} < d_{int}$

Same for $\pi_0 \Rightarrow d_{dec} = 1.8\ 10^{-6}$ m

$t = 4.7\ 10^{-8}\ g/cm^2$

2d. $E = 10^{10}$ eV.

$\beta > \frac{1}{n}$

$\gamma = \frac{10^{10}}{140.10^6} = 71.428$

$\gamma = \frac{1}{(1-\beta^2)^{1/2}} \Rightarrow \beta^2 = \frac{\gamma^2 - 1}{\gamma^2} = 0.99980$

$\beta = 0.99990$

$(n-1) = 2.7\ 10^{-4}$ at sea level $[e^{-10/6.5} = 21\%]$

Density at 10 km = 21% of sea level.

$(n-1)/_{10 km} = 5.8\ 10^{-15}$

$n = 1 + 5.8\ 10^{-5} = 1.000058$

~~$\frac{1}{\beta} = 1.0001$~~ $\frac{1}{n} = 0.99994$

Thus no Cherenkov emission

(5)

2e Photon 5.10^9 eV

Critical energy 80 MeV

Number of steps is $n = 2 \ln \frac{5.10^9}{80.10^6}$

$= \frac{4.135}{0.69} = 6$

Depth after creation is: $6 \times 37 = 222$ g/cm²

Depth at creation is: 221 g/cm²

$X_{max} = 443$ g/cm²

$N = 2^6 = 62,5$ (64)

3a. Hadronic collisions produce pions

$\pi^+ \to \mu^+ + \bar{\nu}_\mu \to e^+ + \nu_e + \bar{\nu}_\mu + \nu_\mu$

$\pi^- \to \mu^- + \bar{\nu}_\mu \to e^- + \nu_e^- + \nu_\mu + \bar{\nu}_\mu$

b At low energies both the pion and the muon decay before they reach ground. The ratio is thus 2.
At higher energies the muon decays on ground mostly. You produce less electron ν; the ratio increases

3 c You will start to deviate from the ratio measured at $\theta = 0$.

This is because of neutrino oscillations:

$$P \sim \sin^2 \frac{\Delta m^2 L}{E} \sin^2 \theta_{mix}$$

If L increases, this changes P.

4 a 1^{st} order 2^{nd} order

b ✓ linear in velocity of ~~cloud~~ shocks

quadratic in velocity clouds

2^{nd} order random ; 1^{st} order shock waves

c e give much stronger synchrotron

p produce π^0 if they collide with gas

~~XX~~ photons → π^0

neutrinos → π^+, π^-

d i) you need to collide with matter to produce π

2) you need magnetic field for synchrotron

The magnetic field lines are twisted

along the jet ⇒ synchrotron emission from electrons

collisions of hadrons occur at the shocks where one produces π^0 π^+, π^- leading to γ and ν.

5 a 1) The star creates heat at the center because of nuclear fusion. This causes radiation pressure

2) The outer layers of the star compress the core by gravitational forces.

There exist hydrostatic equilibrium. If heat in core increases volume and surface expands → more radiation new equilibrium.

If heat generation decreases, the star shrinks, temperature in core rises, new equilibrium sets in.

5 b 1) Temperature in the core controls Maxwell Boltzmann curve, i.e. number of particles as function of energy

2) Probability for fusion is controlled by nuclear charges, reduced mass of nuclei under fusion

5 c 1) A heavy star will have fused the light elements into Fe / Ni.

2) Beyond this mass region fusion will not take place, because energy needs to be supplied (endothermic process)

3) Radiation pressure because of fusion stops, the star will shrink and the electron gas will balance the gravitational pressure. The electron gas acts instead of the radiation field.

4) If the energy of the electrons is high they can be absorbed into protons (inverse beta-decay).
The electron gas pressure drops, the star undergoes a gravitational collapse.